JOURNAL OF AIRCRAFT
Vol. 40, No. 2, March–April 2003

Vibration Control Using Fuzzy-Logic-Based Active Damping

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In this study a general approach is introduced for the design of a robust control law for suppression of structure borne vibration. This control law is based on a passive design in the form of dynamic vibration absorbers. Passive absorbers minimize vibration at a specific frequency, but their performance is improved by introducing adaptive tuning of the absorber. An adaptive dynamic vibration absorber is tuned to the forcing frequency, using classical methods. The tuning ratio is time varying and adapts itself to variations in the forcing frequency. However, the uniqueness of the approach in this study is that the damping parameter of the absorber is continuously varied by means of a fuzzy-logic control algorithm to provide a lower sound pressure level. The inputs of the fuzzy control law are the displacement and velocity of the main structure. The effectiveness of the control algorithm for active vibration control is demonstrated using MATLAB® simulations of a single-degree-of-freedom plant. This methodology provides superior performance in the presence of significant mistuning compared to a more conventional approach.

I. Introduction

A MAJOR issue in the cabin design of commercial transport aircraft concerns the reduction of noise for improved passenger comfort. Cabin noise usually results from either airborne sources, such as engine fan, propeller tones, propwash or engine exhaust noise, or from structure-borne sources such as engine spool imbalance. In addition to cabin noise, the preceding disturbances can also cause material fatigue. The sound pressure level can be attenuated by the incorporation of structural acoustic control.

Structural acoustic control can be achieved by using a passive design like the passive dynamic vibration absorber (DVA). This device typically consists of a second-order dynamic system comprising of mass, spring, and dash-pot elements (Fig. 1), whose main purpose is to transfer and dissipate the energy of the system, thereby reducing the sound pressure level. George[1] reports on the introduction of DVAs for sound suppression in C90B and King Air B2000 business aircraft. The DVAs, tuned to the low-frequency structural vibration that results from the turboprop engines, provided 17-dB noise reduction in the center of the C90B cabin. The main drawback of this approach is that the DVA is effective only in the immediate vicinity of its tuned frequency, whereas in practice there are fluctuations in the frequency of vibration that can result from the throttling of the engines.

The controller proposed by Ryan[2] acts to alter the stiffness of the adaptive absorber (see schematic configuration in Fig. 2). The accelerometer mounted on the main system mass provides an ac signal that is fed through a frequency-to-voltage converter. The dc output of the converter, which is proportional to the frequency of the input signal, yields the desired tuned frequency of the absorber. The desired stiffness k_2 is then determined from the value of the desired tuned frequency. The error between the desired stiffness and the actual stiffness is then fed back to alter the length of a variable length cantilever-type absorber that utilizes a length change in the beam to alter the absorber stiffness. More details of the actuation mechanism are provided by Ryan.[2]

The case for a tunable passive vibration absorber has led to hardware solutions such as a discrete stiffness spring proposed by Walsh and Lamancusa.[3] A promising hardware design, introduced by Davis and Lesieutre,[4] incorporates a shunted piezoceramic inertial actuator. The electrical tuning of this absorber, which changes the mass or stiffness of the device, is enabled by the piezoelectromechanical coupling. The electromechanical properties of the piezoceramic forcing element within the adaptive absorber in conjunction with an external passive electrical shunt circuit can be used to alter the natural frequency and damping of the device. The natural frequency of the device can be altered by capacitive shunting, whereas resistive shunting alters both the natural frequency and damping.

II. Objective of This Study

This research effort addresses the incorporation of an adaptive vibration absorber in which continuous tuning of the damping parameter of the absorber is achieved by a fuzzy-logic control (FLC) algorithm. The main objective of this study is to determine the effectiveness of the developed methodology, based on numerical simulations of an experimental model used to test adaptive absorbers.[2] The performance robustness and the closed-loop performance of the developed approach are examined by comparing the simulation results with those obtained using the approach suggested by Ryan.[2] The main reasons for selecting variable damping and fuzzy logic are also explained.

The current effort does not go into the details of hardware implementation. A schematic description of the proposed configuration is provided in Fig. 3. The displacement and the velocity of the main system mass are measured. Based on these two sensor readings, the desired values for the variable stiffness k_2 and the variable damping coefficient c_2 are calculated as follows:

1) The forcing frequency is extracted from the sensor readings, and, using Den Hartog's tuning scheme described next, the desired value of k_2 is calculated.

2) The sensor readings are fed into the adaptive fuzzy control algorithm (AFCA) as inputs. The output of this algorithm is the desired value of the variable damper c_2.

Finally the errors between the desired and the actual value of the preceding design variables are fed back to direct the respective

Received 9 February 2001; revision received 22 July 2002; accepted for publication 1 August 2002.

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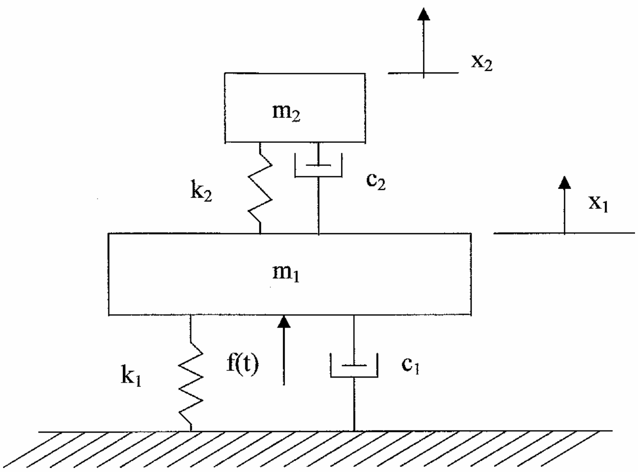


Fig. 1 Plant with attached vibration absorber.

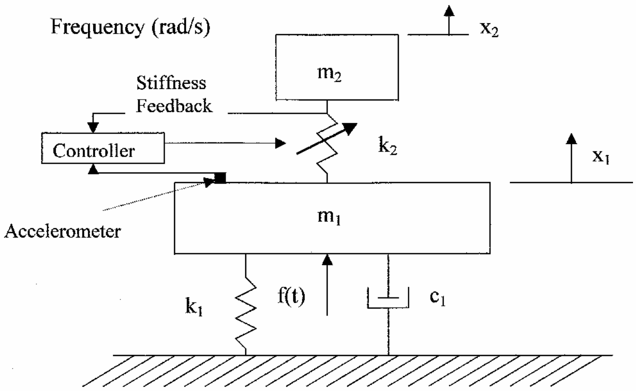


Fig. 2 Adaptive vibration absorber (no damping).

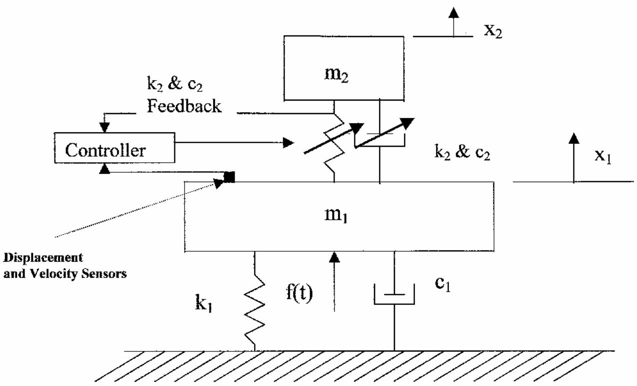


Fig. 3 Adaptive FLC vibration absorber.

actuators. A possible hardware solution for variable stiffness and damping absorbers, based on piezoceramic materials, is detailed by Davis and Lesieutre.[4]

III. Time Variant Damping

The optimal tuning and damping ratios for an absorber attached to a primary structure are formulated by Den Hartog.[5] The best strategy for specifying the damping to be introduced was constrained by the requirement for a linear time-invariant system. Lifting this self-imposed constraint, Shahruz et al.[6] showed that the optimal damping ratio for linear second-order systems which results in a minimum-time response to step inputs was of bang-bang type, that is, the damping ratio switches between its minimal and maximal values at certain switch points.

The realization that variable damping provides enhanced performance has not been considered seriously for the control of flexible structures for a wide variety of reasons. Some of them are 1) lack of robustness in view of uncertainties in plant model and external noise, 2) properties of the switch points (how many and when they occur) depend on the character of the transient disturbance, that is, sensitivity to initial conditions, and 3) implementation into a closed-loop system is seldom practically effective.

The incorporation of optimal variable damping requires an approach that enables the integration of the control law into a flexible structure with relative ease and simplicity while providing the required robustness characteristics. In the present effort an approach based on fuzzy logic is introduced to achieve the desired control. In this approach the varying of the damping ratio is smooth, that is, the damping does not jump from minimal to maximal values, and it is naturally determined in closed loop. As will be shown next, the FLC variable damping approach seems to exhibit excellent robustness characteristics without sacrificing nominal plant performance.

IV. Fuzzy-Logic Control

Fuzzy logic, which is the logic on which fuzzy control is based, is a convenient way to map an input space into an output space.[7] The logical system that captures the spirit of our approximate, imprecise world was introduced by Zadeh[8] as the theory of fuzzy sets, which in time proved to be a very powerful tool for dealing quickly and efficiently with imprecision and nonlinearity. The experience of the past decade, with the successful marketing of a wide variety of products based on the FLC,[9] has shown that for certain applications use of FLC can lead to lower development costs, superior features, and better end-product performance. One of the inherent properties of fuzzy-logic systems is that it has the capability of being a universal approximator. This implies that by using adequate inputs, and a number of rules and fuzzy sets for each input variable, a fuzzy-based system can approximate any real continuous nonlinear function to an acceptable degree of accuracy.[9] The implementation of a variable damping strategy requires such a universal approximator that can successfully emulate the bang-bang type of minimum-time control.

The capability of FLC to emulate time-optimal bang-bang control was attributed, by Thomas and Armstrong-Hélouvry,[10] to what is termed as the generalized damping benefit of FLC, which can provide fast and effective system responses. For large values of system error, the damping effect of the error derivative control is blocked as full control authority is used to quickly drive the system error to zero. As the system error tends to zero, a progressively greater damping effect is introduced. This nonlinear approach is in complete contrast to the tradeoff required between the rise time, overshoot, and control effort seen in linear control. Another shortcoming of linear control is that it is far from time optimal when control authority is bounded.[11]

The controller proposed by Cohen et al.[12] provided continuous tuning of the damping parameter of the absorber just described by FLC. Its parameters could be adapted to provide fairly fast control for large deviations, of the measured state of the plant from the desired state, and a minor amount of control for small deviations. Cohen has applied this methodology to several benchmark problems using MATLAB® simulations.[13]

V. Closed-Loop Dynamic Model

A two-degree-of-freedom second-order dynamic system, presented in Fig. 1, describes a primary structure with mass m_1, stiffness k_1, and damping coefficient c_1. A secondary mass m_2 is attached to the main mass by stiffness k_2 and damping coefficient c_2. The main mass is harmonically excited by a sinusoidal force $f(t) = F \sin(\Omega t)$. The resulting equations of motion for the case of steady-state excitation can be written as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F \sin(\Omega t) \\ 0 \end{bmatrix} \tag{1}$$

Table 1 Fuzzy logic rule base

Inputs → ↓	Negative $x_1(t)$	Small negative $x_1(t)$	Zero $x_1(t)$	Small positive $x_1(t)$	Positive $x_1(t)$
Positive $dx_1(t)/dt$	Very small	Small	Small	Small	Very small
Small Positive $dx_1(t)/dt$	Very small	Small	Large	Small	Very small
Zero $dx_1(t)/dt$	Small	Medium	Very large	Medium	Small
Small negative $dx_1(t)/dt$	Very small	Small	Large	Small	Very small
Negative $dx_1(t)/dt$	Very small	Small	Small	Small	Very small

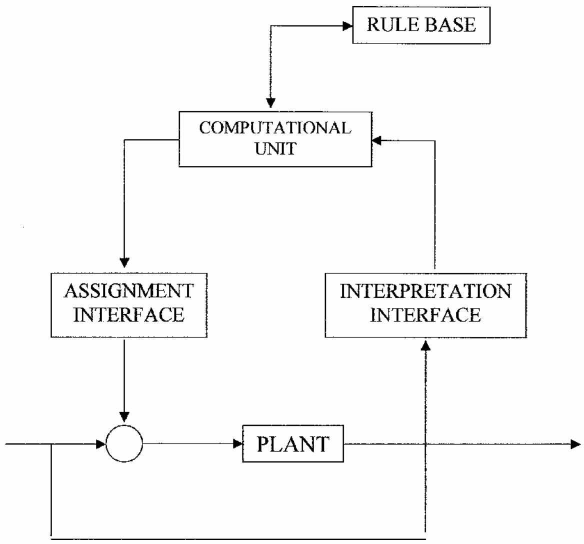


Fig. 4 Fuzzy-logic control system.

From Eq. (1) the steady-state amplitude of the main mass displacement x_1 has a magnitude given by

$$|x_1| = F \cdot \frac{\sqrt{\left(k_2 - m_2\Omega^2\right)^2 + (c_2\Omega)^2}}{\sqrt{(A^2 + B^2)}}$$

$$A = (m_1 m_2)\Omega^4 - [c_1c_2 + m_2(k_1 + k_2) + m_1k_2]\Omega^2 + k_1k_2$$

$$B = (c_1k_2 + c_2k_1)\Omega - [c_2(m_1 + m_2) + c_1m_2]\Omega^3 \qquad (2)$$

Based on Den Hartog's[5] approach, the steady-state amplitude of the main mass can be minimized to zero for the special case when the absorber is undamped ($c_2 = 0$), and its natural frequency is tuned to the forcing frequency as shown in Eq. (3):

$$\Omega_{\text{abs}} = \Omega = \sqrt{k_2/m_2} \qquad (3)$$

From Eq. (2) it can be deduced that an undamped absorber is effective in absorbing vibrations forced at its fundamental frequency.

In practice, however, it is rarely possible to exactly tune the absorber to the forcing frequency, which fluctuates. Therefore, it is common practice to introduce damping into the absorber in order to obtain an acceptable main system response within a bandwidth about a nominal excitation frequency. Here, damping is introduced into the absorber using a variable damping FLC based on the algorithm developed by Cohen et al.[12] This algorithm has been very effective in several cases, providing fast response to initial conditions and robust performance, and therefore was chosen here in order to examine its potential effectiveness to the case of forcing frequency.

The major mechanisms of the FLC are 1) a set of if-then statements called linguistic control rules and 2) a fuzzy inference system that interprets the values in the input vector and, based on the linguistic rules, assigns values to the output vector. The structure of a fuzzy-logic controller is depicted in Fig. 4. The first stage in building the fuzzy part of the controller, described in Fig. 4, is referred to as "fuzzification" of the input/output parameters. The inputs of the algorithm are the displacement x_1 and the velocity $\dot{x}_1$ of the primary mass m_1, whereas the output of the fuzzy-logic-based algorithm is the damping coefficient of the absorber c_2.

Five membership functions, namely, POSITIVE, SMALL POSITIVE, ZERO, SMALL NEGATIVE, and NEGATIVE, are used to describe each of the input parameters (i.e., displacement and velocity of primary mass). In addition, the output parameter (i.e., damping coefficient of absorber) is also described using five membership functions, namely, VERY LARGE, LARGE, MEDIUM, SMALL, and VERY SMALL. The respective membership functions for the input/output parameters are obtained based on a tuning process as described in Cohen.[13] The fuzzy adaptation strategy, presented in this effort, is based on rules of the form "if... then...," which convert inputs (normalized transverse displacement and velocity) to a single output (actuation command), that is, conversion of one fuzzy set into another.[9] Heuristic rules based on well-experienced structural insight are coupled with fuzzy reasoning whereby large values of the inputs require a lightly damped absorber, which would provide quick rise times. However, when the plant state is in the vicinity of the desired state, the damping factor is large to reduce the overshoot and steady-state error. The rule base, presented in Table 1, describes a set of 25 rules. A typical rule can be read as follows:

If $x_1(t)$ is negative and $\dot{x}_1(t)$ is positive, then $c_2(t)$ is very small

Abihana[14] defines inference as the process of applying the degree of membership, computed for a production rule premise, to the rule's conclusion to determine the action to be taken. The value assigned to the output can either be scaled (max-dot method) or clipped (max-min) to the degree of membership of the premise. Both of these methods provide similar results. The preceding inference methods are the most common methods used in fuzzy-logic control. Nevertheless, several additional methods exist, as described by Wang.[15]

As observed in Table 1, the rule base contains quite a few rules relating to the same output variable. Therefore, to obtain an overall output in the fuzzy state, an inference method is applied. First, the degree of fulfillment of each and every rule is found by applying the fuzzy "AND" operation. Let us represent the individual elements of the rule-base "matrix," presented in Table 1, as

$$\delta_{ij}(i = 1, 5; \quad j = 1, 5)$$

where

$$\delta_{ij} = \text{Minimum}(\mu_Q, \mu_L) \qquad (4)$$

where μ_Q represents the membership functions of displacement $x_1(t)$, and μ_L represents the membership functions of velocity $dx_1(t)/dt$, for Q = Positive, Positive Small, Zero, Negative Small, and Negative and L = Negative, Negative Small, Zero, Positive Small, and Positive.

In the next step all of the output values, obtained by clipping or scaling, are then brought together to form the final output membership function. After evaluation of the propositions, the output values represented are unified to produce a fuzzy set incorporating the solution variable. This unification of outputs of each rule, referred to as aggregation, occurs only once for each output variable. The aggregation process, always comprised of a commutative method, can be any one of the methods as described by Jang and Gulley[16]: MAX (maximum), PROBOR (probabilistc or), and SUM (simply the sum of each rule's output set). In this effort the method applied is the Bounded SUM (simply the sum of each rule's output set having an upper bound of 1). Applying the sum to the rule base given Table 1, the union of the fuzzy sets for the same output variable is taken to reach the respective aggregation of the output. The rule base is usually not made to be part of the tuning process. However, the sensitivity of closed-loop performance to changes in the rule base is examined, and minor changes are made in order to ensure the desired performance.

Finally, to achieve a practical controller a control action comprised of a single numerical value is required. Therefore, the space of the fuzzy damping factor, obtained using the method described in the preceding section, is mapped into a nonfuzzy space (crisp) in a process known as "defuzzification". There are various strategies aimed at producing a crisp value. Some of the commonly used strategies are the center of area (COA), the mean of maximum, and the max criterion.[17] However, there is no accepted systematic methodology for selecting a defuzzification strategy. Herein, the COA scheme is adapted. This strategy was found to yield better steady-state performance when compared to the other strategies just mentioned.[17] Cohen et al.[18] has incorporated the COA method within the developed fuzzy-based algorithm for cantilever beam vibration control using piezoceramic materials.

Actual implementation issues, such as the of speed of adaptation, are considered here.

VI. Illustration Example

To appreciate the potential of the developed strategy, a computational example is presented. The parameters of the example, depicted in Table 2, are identical to those used by Ryan.[2] The main objective is to control the steady-state amplitude of the primary mass displacement rms, defined as:

$$RMS = \sqrt{\sum_{i=1}^{N} x_i^2 \Big/ N} \tag{5}$$

where x_i is the displacement of primary system mass m_1 at the ith time step and N the number of time steps in the simulation.

Table 2 Parameters for example plant

Parameter	Value
m_1	2.446 kg
m_2	0.2446 kg
K_1	1921 N/m
K_2	Variable
c_1	60.0 Ns/m
c_2	Variable
Nominal operating frequency	28.0 rad/s
Force amplitude	200 N

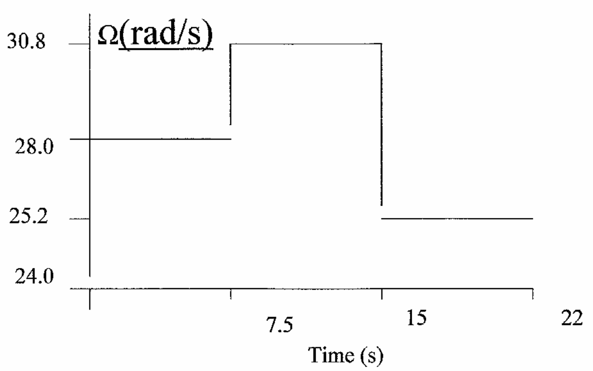


Fig. 5 Mission profile (frequency fluctuations) used in perturbed plant simulations.

To illustrate the decibel reduction between example comparisons for the ith time step, the following relation is used[2]:

$$\text{dB} = 20 \cdot \log_{10}\left[\frac{RMS}{RMS_{\text{uncontrolled}}}\right] \tag{6}$$

The frequency profile used in simulations to examine the effect of the fluctuations in the frequency of the vibration is defined in Fig. 5. The system is initially at rest, and the controller is activated at $t = 0$ s. At $t = 7.5$ s the driving frequency increases by 10% of the nominal to 30.8 rad/s. At time $t = 15$ s the driving frequency decreases by 10% of the nominal to 25.2 rad/s. The simulations end at $t = 22.5$ s. Two sets of simulations were run on a MATLAB® platform. The first set represented the Nominal System, where the temporal plant dynamics is well known. The second set of simulations concerns performance robustness testing based on two perturbed plants, obtained by increasing/decreasing m_2 by 10%. The simulations based on the nominal system are conducted for four cases, namely, 1) Case 0, the uncontrolled structure; 2) Case 1, passive vibration absorber (PVA); 3) Case 2, active vibration absorber (AVA) using the method suggested by Ryan,[2] whereby k_2 is adaptive to the forcing frequency; and 4) Case 3, active-controlled DVA using the strategy developed here, whereby k_2 is adaptive to the forcing frequency and c_2 is varied using an AFCA.

VII. Results and Discussion

The results of the first set of simulations are presented for the known nominal plant in Table 3. The salient observations made from this table are as follows:

1) The simulation successfully reproduced the results presented in Ryan.[2]

2) The introduction of AVA substantially improves the performance of the system compared to the PVA case.

3) The introduction of variable damping using AFCA does not impair the closed performance for the nominal system.

Time response of x_1 is given in Fig. 6a for the uncontrolled plant. Figure 6b presents the simulation results for AFCA. (The results obtained for AVA are practically identical.) The behavior of k_2 and c_2 is given in Fig. 7.

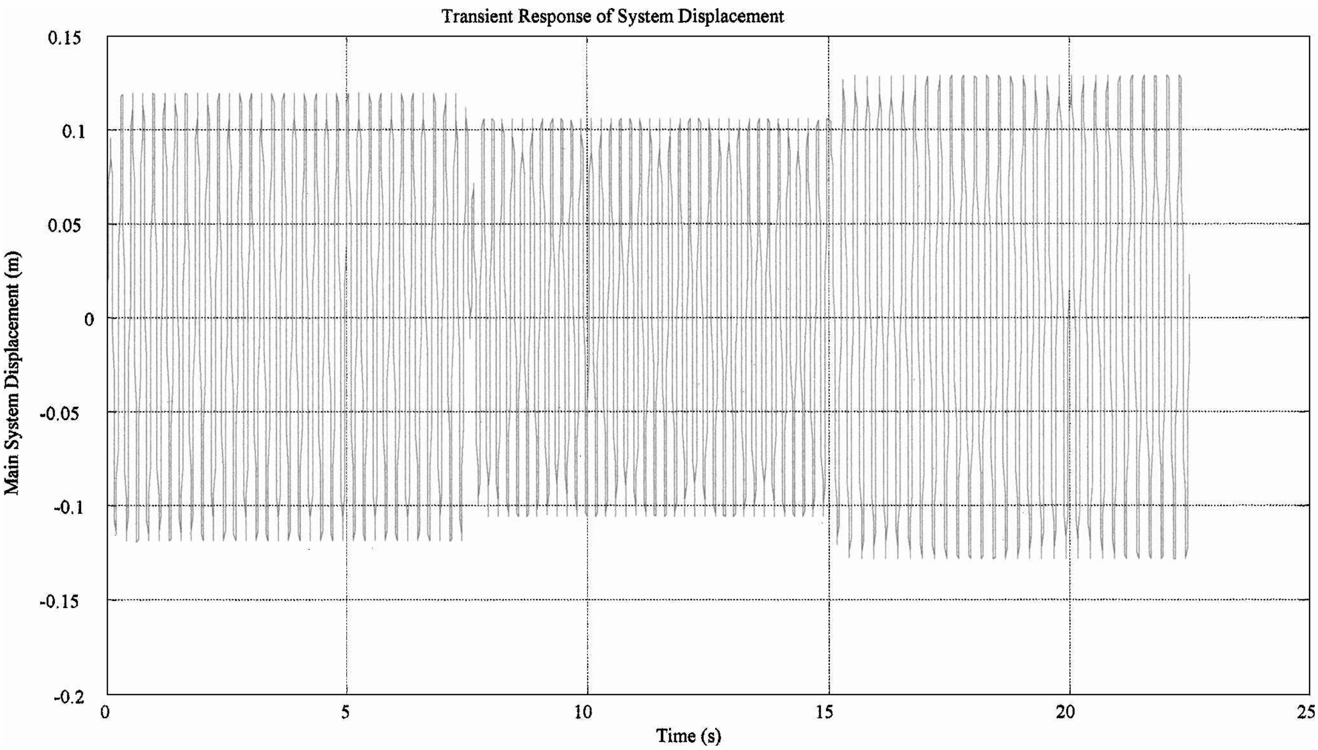
The results of the second set of simulations, based on the perturbed plants, are presented in Table 4. The salient observations made from this table are as follows:

Table 3 Comparisons of the effectiveness of the vibration absorber for the nominal plant

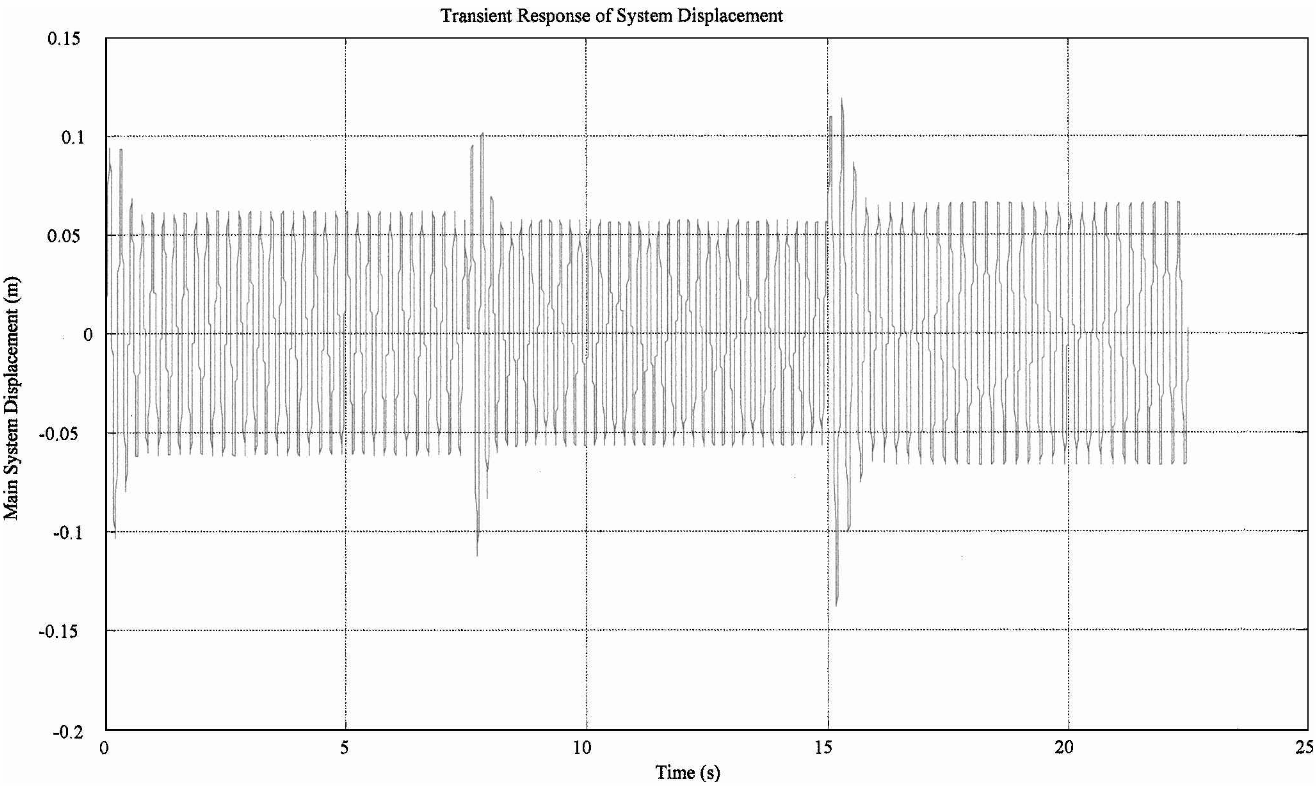
Case	RMS displacement, m	Reduction from PVA, %	Reduction from PVA, dB
Case 0, Uncontrolled	0.085	——	——
Case 1, PVA	0.064	——	——
Case 2, AVA with no damping	0.028	56	7
Case 3, active vibration absorber with FLC for variable damping	0.028	56	7

Table 4 Comparison of the effectiveness of the vibration absorber for the perturbed plants

Case	RMS displacement, m	Reduction from PVA, %	Reduction from PVA, dB
Nominal plant			
AVA	0.028	56	7
AFCA	0.028	56	7
Perturbed plant A (m_2 increased 10%)			
AVA	0.058	10	1
AFCA	0.026	59	8
Perturbed system B (m_2 decreased 10%)			
AVA	0.061	4	0.4
AFCA	0.029	55	7



a)



b)

Fig. 6 Time response uncontrolled and AFCA.

1) AVA is extremely sensitive to uncertainties in the dynamic model of the plant, and the performance of the active absorber degrades significantly when the plant is perturbed. In fact, the performance is only slightly better than that of the PVA.

2) On the other hand, the AFCA displays excellent robustness and maintains the same level of performance when the plant is perturbed.

An essential requirement from a flexible structure controller constitutes robustness.[13] Indeed AVA performs well in an ideal situation (full knowledge of plant model and no measurement noise); however, it is not a good choice at a real plant. Modeling uncertainties and noise lead to inaccurate switching of the absorber stiffness, and this leads to a performance that is very close to that obtained by PVA. In such a case, the added cost of going from passive to active might not be justified.

At this stage one cannot help wondering as to whether Ryan[2] would have obtained augmented robustness if his approach were extended by introducing damping into the absorber, that is, $c_2 \neq 0$, which can be obtained by an external passive electrical shunt circuit.

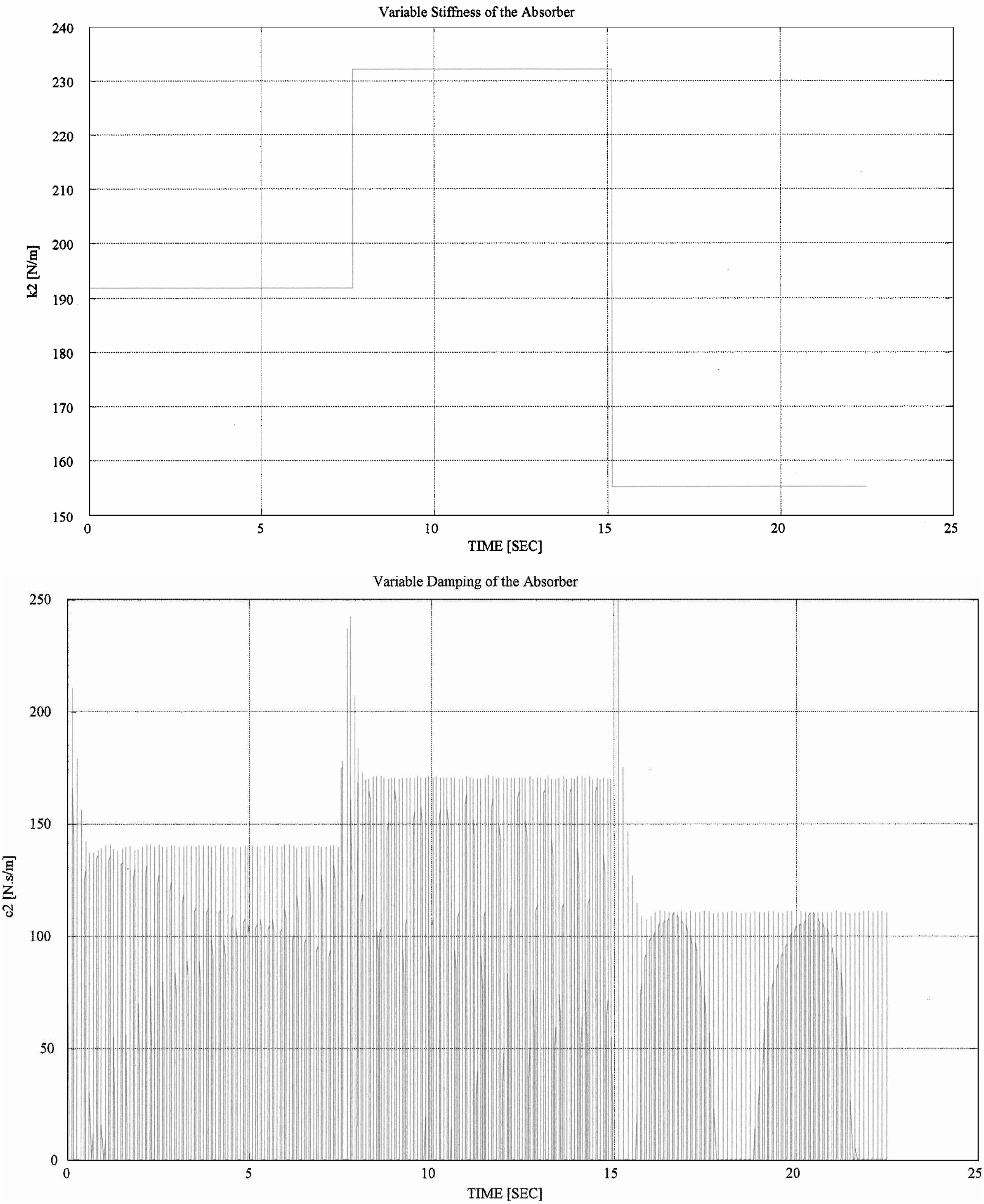


Fig. 7 Stiffness and damping for AFCA.

Another question that crosses the mind is whether the added effort associated with the implementation of AFCA is justified, that is, the effectiveness of the variable damping strategy. These questions were addressed by running several simulations of the nominal and perturbed plants for different values of c_2. The results of these simulations are summarized in Table 5. Examination of the results depicted in this table emphasizes the merits of the proposed approach based on AFCA. For the given mission profile[2] and a fixed and nonzero absorber damping, lowest RMS values are obtained for c_2 in the vicinity of 0.5. It is obvious that this value depends on the nature of the mission profile (forcing frequencies). It is also noticed that there is a tradeoff between the result for the nominal plant ($c_2 = 0$) and robustness for perturbed plants (larger values of c_2). A variable damping approach seems to exhibit excellent robustness characteristics without sacrificing nominal plant performance.

Table 5 Effect of constant damping on effectiveness of AVAs compared with that for variable damping

Run no.	Damping coefficient ζ	Nominal plant RMS, m	Perturbed plant A RMS, m	Perturbed plant RMS, m
1	0.00	0.0275	0.0577	0.0614
2	0.25	0.0304	0.0503	0.0557
3	0.40	0.0344	0.0490	0.0546
4	0.45	0.0357	0.0488	0.0545
5	0.50	0.0370	0.0487	0.0514
6	0.55	0.0383	0.0487	0.0543
7	0.60	0.0396	0.0489	0.0544
8	1.00	0.0475	0.0513	0.0562
9	2.30	0.0601	0.0603	0.0626
10	Varied [0,7.5]	0.0275	0.0263	0.0288

VIII. Conclusions

An adaptive vibration absorber, based on variable stiffness and damping characteristics, is proposed for the suppression of sound induced by a vibrating structure experiencing fluctuations in the frequency of the forced vibration. The damping of the absorber is continuously varied on the basis of a fuzzy-logic algorithm. The inputs of the control law are the displacement and the velocity of the main structure.

When compared to an adaptive absorber based on variable stiffness alone, the proposed strategy provides similar closed-loop performance (nominal profile) and superior robustness characteristics (perturbed profile). The phenomenon associated with the benefits of variable damping can be further examined with the aim of obtaining an optimal strategy.

The benefits of variable damping can be verified experimentally using a smart structure, based on piezoceramic sensors/actuators. The proposed strategy can be extended for application on flexible structures such as beams, plates, and shells.

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